Chapter 26
Scientific Rationality by Degrees

Alexandru Marcoci and James Nguyen

Abstract In a recent paper, Okasha imports Arrow's impossibility theorem into the context of theory choice. He shows that there is no function (satisfying certain desirable conditions) from profiles of preference rankings over competing theories, models or hypotheses provided by scientific virtues to a single all-things-considered ranking. This is a *prima facie* threat to the rationality of theory choice. In this paper we show this threat relies on an all-or-nothing understanding of scientific rationality and articulate instead a notion of rationality by degrees. The move from all-or-nothing rationality to rationality by degrees will allow us to argue that theory choice can be rational *enough*.

Keywords Theory Choice • Kuhn • Okasha • Social Choice Theory • Rationality

26.1 Introduction

Imagine a scientist, or group thereof, who cares about multiple scientific virtues – accuracy, simplicity and scope for example – and who is faced with a set of competing theories, models or hypotheses. How is she to choose the most virtuous of the alternatives, all-things-considered? How is she to rationally choose the 'best' competitor? In a recent paper, Okasha (2011) provides an argument that seriously

A. Marcoci (✉)
Department of Government, London School of Economics and Political Science

Department of Philosophy, Logic and Scientific Method, London School of Economics and Political Science, London, UK
e-mail: A.Marcoci@lse.ac.uk

J. Nguyen
History and Philosophy of Science, University of Notre Dame, Notre Dame, IN, USA

Centre for Philosophy of Natural and Social Science, London School of Economics and Political Science, London, UK
e-mail: J.Nguyen4@nd.edu

© Springer International Publishing AG 2017

M. Massimi et al. (eds.), *EPSA15 Selected Papers*, European Studies in Philosophy of Science 5, DOI 10.1007/978-3-319-53730-6_26

a.marcoci@lse.ac.uk

threatens the possibility of such a procedure. His argument is inspired by Kuhn's (1972) claim that there is no *unique* theory choice algorithm. According to Kuhn, when faced with a set of alternatives even scientists who agreed on what virtues are important could still rationality disagree regrading which alternative is the best, all-things-considered. But rather than arguing that there is no unique theory choice algorithm, Okasha utilises formal tools from social choice theory to argue that there is *no* such algorithm whatsoever.

In this paper we draw attention to Okasha's assumption that scientific rationality is an all-or-nothing notion and motivate the move to a degrees notion of rationality, instead. By considering rationality in *degrees*, rather than in the all-or-nothing sense, we can precisely gauge the threat Okasha poses, and we show that this is highly sensitive to the number of alternatives and virtues under consideration. Whether or not theory choice is 'rational' depends on where one sets a threshold. This a substantial decision and different scientists may reasonably disagree. We do not offer a solution to this problem, nor do we believe there is one. However, we argue the constraint on this threshold is sufficiently weak in order to make it reasonable to accept that choosing among competing theories based on their relative ranking according to a set of scientific virtues is rational *enough*.

26.2 The Impossibility Result

Okasha's strategy is remarkably simple. The m scientific virtues, which form a set $\mathfrak{V}$, are treated as voters. The n competing theories provide a domain of alternatives $\mathfrak{T}$, and each virtue provides an ordinal ranking defined over the competitors, from most to least virtuous. These rankings can then be treated as preference rankings: transitive, reflexive, and complete binary relations defined over $\mathfrak{T}$.[1] When a virtue i prefers a theory x to a theory y we write $y \prec_i x$. Given virtues 1 through m (in $\mathfrak{V}$), a profile is an ordered tuple of rankings. A theory choice function takes profiles as arguments and delivers an all-things-considered binary relation $\preceq$, defined over $\mathfrak{T}$. A theory x is preferred (all-things-considered) to theory y if and only if $y \preceq x$ and it's not the case that $x \preceq y$.

What conditions should a theory choice function satisfy in order for it to be considered rational? (Okasha 2011,92–93) points out that all five of Arrow's social choice conditions have analogues that are intuitively desirable in the context of theory choice. Unrestricted domain (UD) stipulates that theory choice is possible irrespective of how the given competing theories are ranked by virtues. Weak Pareto (WP) requires that if all virtues rank theory x over theory y then $y \prec x$. According to Independence of Irrelevant Alternatives (IIA) when ranking two theories the

[1]For the purposes of this paper we treat preference rankings as strict (asymmetric) relations. This is a standard simplifying assumption made in the social choice literature, and where relevant our results can be modified accordingly to apply to non-strict rankings of theories by virtues. This is of no conceptual importance.

a.marcoci@lse.ac.uk

all-things-considered preference ranking should only take into consideration how the two theories are ranked by the individual virtues. No information about the relative rankings of a third theory should be relevant. Non-Dictatorship (ND) is the condition that none of the virtues are such that whenever they prefer x to y (for all x, y and any way the other virtues rank the two), x is preferred to y in the all-things-considered ranking. Such a virtue would act as a 'dictator'. A fifth condition is that the theory choice function always delivers a transitive, complete preference ordering. We call this Overall rationality (OR).[2] Since the formal structure of ranking theories according to virtues, and social alternatives according to voters, is the same, Arrow's impossibility result immediately applies (Arrow 1951). There is no theory choice function that satisfies UD, WP, IIA, ND and OR. As Okasha notes, if the rationality of theory choice is identified with the existence of such a function then the result demonstrates that rational theory choice is impossible.

There has been much recent discussion about how to escape the conclusion of Okasha's argument. Okasha himself suggests that if the informational basis of scientific virtues is enriched (in the simplest case by scoring the theories on a common cardinal scale), then the impossibility result is avoided. This amounts to rejecting IIA in the context of theory choice and is discussed in more detail in Rizza (2013) and Stegenga (2015). Similarly Gaertner and Wüthrich (2016) suggest imposing a cardinality via a scoring rule, although they argue that a cardinal version of IIA is retained in their framework. Okasha also considers weakening OR by demanding that the theory choice function deliver only a *best* all-things-considered theory, as opposed to a ranking (cf. Footnote 2). Relatedly, Bradley (2016) argues that we should lower our expectations of what rationality requires. Rather than demanding a transitive, reflexive and complete ranking of theories, rationality only *rules out* certain choices. Alternatively, Morreau (2014) suggests restricting UD.

But rather than questioning any of the conditions, our strategy in this paper is to investigate alternative ways of construing theory choice as a rational enterprise which don't require identifying rational theory choice with the existence of a function that satisfies the five Arrovian conditions. We do this by focusing on the extent to which pairwise majority voting is rational.

[2]Note that usual expositions of Arrow's result, Okasha's included, do not include OR as part of the conditions leading to the inconsistency, but rather build it into the definition of an aggregation function. We decide to do so since we are interested in the behavior of functions that output non-transitive relations, and moreover it will be useful to gauge the threat posed by Arrow's impossibility under OR as opposed to the threat under a natural weakening of it. This weakening is the requirement that the aggregation function deliver a winner, instead of a complete preference ordering. The motivation for this is simple in our theory choice context: after all, according to Kuhn, the winners are those who rewrite the history of science. Where appropriate we refer to this weakened notion with the prefix Condorcet rationality.

a.marcoci@lse.ac.uk

26.3 Minimal Rationality

In this section we introduce a minimal all-or-nothing notion of rationality, call it 'minimal rationality'. This is the building block which will be used to construct the notion of rationality by degrees that interests us in this paper. Begin with a social choice example. Suppose Albert, Bill, and Chloe are trying to choose, as a group, between watching a football match, going to the cinema, or visiting a restaurant. Suppose they have the following preference rankings:

$$\begin{aligned} &\textbf{Albert}: \textit{Football} \prec_A \textit{Cinema} \quad\ \ \prec_A \textit{Restaurant} \\ &\textbf{Bill}: \quad\ \textit{Football} \prec_B \textit{Cinema} \quad\ \ \prec_B \textit{Restaurant} \\ &\textbf{Chloe}: \ \textit{Football} \prec_C \textit{Restaurant} \prec_C \textit{Cinema} \end{aligned}$$

In this social choice context, the Arrovian conditions are supposed to supply constraints on what the group should do. Some of these conditions, such as IIA for example, put inter-profile constraints on the behaviour of an aggregation function. Others, such as OR, put constraints on the behaviour of a function that apply profile by profile. What we call 'minimal rationality' supplies a way of thinking about whether an aggregation function that satisfies the Arrovian conditions of WP, IIA, ND and UD, e.g. pairwise majority voting as we define it below, is normatively acceptable on a profile by profile basis. The conditions on this are those supplied by OR: the value of the function when applied to that profile should itself be a preference ranking, i.e. transitive and complete. Our point then, is that some aggregation functions can be normatively acceptable at some profiles, and yet normatively unacceptable at others.

At the profile displayed above pairwise majority vote supplies the following:

$$\textbf{Group}: \textit{Football} \prec \textit{Cinema} \prec \textit{Restaurant}$$

and is thus normatively acceptable with respect to this profile. However in a scenario where Albert, Bill, and Chloe held preference rankings such that pairwise majority voting delivers an intransitive all-things-considered value, then the function is not normatively acceptable with respect to that profile. This would be the case if they had held preference rankings that generated Condorcet's paradox. A weaker requirement that might be of particular relevance in the context of theory choice is that the function deliver a Condorcet winner (an alternative preferred to all other alternatives in the all-things-considered ranking), rather than a preference ranking. With three alternatives, these conditions are equivalent.

When applying the machinery of social choice theory to theory choice, Okasha's strategy is to take the question of whether or not an aggregation function is normatively acceptable to correspond to the question of whether or not a (theory choice) aggregation function is rational. And he takes the conditions on whether or not this is the case to be the Arrovian ones. Again, some of these conditions put inter-profile constraints on the behaviour of such functions, and others apply profile-

a.marcoci@lse.ac.uk

by-profile. So again, we can ask of a given aggregation function which satisfies the conditions of WP, IIA, ND and UD whether or not it is 'minimally rational' on a profile-by-profile basis. And again, the conditions on this are supplied by OR. These observations provide the following definition:

Minimal rationality A theory choice function f is minimally rational with respect to a profile $\mathcal{P} \in \mathfrak{D}^n_m$, if and only if it meets UD, WP, IIA, ND (with respect to $\mathfrak{D}^n_m$) and takes $\mathcal{P}$ to a transitive and complete ranking.[3]

A natural weakening of minimal rationality is to demand a Condorcet winner, rather than a transitive complete ranking. This gives rise to an analogous notion of minimal Condorcet rationality in the obvious way.

Minimal rationality can then be built up to the full blown notion of rationality that Okasha requires, in the sense of meeting all of the Arrovian conditions everywhere in a domain of profiles. If a function f is minimally rational for every profile in $\mathfrak{D}^n_m$ then choosing the most suitable theory out of n alternatives using m virtues by means of f is always rational. In other words, there is nothing more to being rational with respect to a domain than being rational with respect to every element in that domain. This is the requirement Okasha argues is not met by any theory choice function. Indeed, by Arrow's theorem there is no f satisfying this (for any $n \geq 3$ and $m \geq 2$). In Sect. 26.5 we discuss a weakening of this requirement and show that with it in place, the prospects of rational theory choice improve.

26.4 Fixing the Theory Choice Function

One thing to note before introducing the notion of rationality by degrees is that in this paper we restrict our attention to a particular theory choice function. Arrow's result is general, it entails that there is *no* function that is rational for domains where $n \geq 3, m \geq 2$. As such, discussion of any particular function can be suppressed. This is not so when discussing minimal rationality and rationality by degrees. Therefore for the purposes of this paper we focus on pairwise majority voting, which is defined as follows:

Pairwise majority voting For a set of virtues $\mathfrak{V}$, let $\Delta^+ =_{df} \{i \in \mathfrak{V} : y \prec_i x\}$, $\Delta^- =_{df} \{i \in \mathfrak{V} : x \prec_i y\}$. Then: y $\prec$ x if and only if $|\Delta^+| \geq |\Delta^-|$.[4]

There are other functions that satisfy UD, WP, IIA, and ND, including Pareto dominance and extension procedures, both of which violate completeness for various profiles, see (List 2013,§3.2.2). We nevertheless ignore these other functions for the remainder of this paper. Our purpose is to demonstrate how important one's

[3] $\mathfrak{D}^n_m$ represents the class of all profiles that can be defined over m virtues and n theories.

[4] For the remainder of this paper we will assume there is an odd number of virtues. This means that the output of the aggregation, under pairwise majority, will always be a strict ordering (if an ordering, at all).

a.marcoci@lse.ac.uk

construal of scientific rationality is when evaluating rational theory choice, rather than determining which aggregation function is most suitable for theory choice. For this purpose it is enough to look at a single aggregation method and it seems natural to use the most well-studied one for our proof of concept.

26.5 Rationality by Degrees

In Sect. 26.3 it was noted that it is not always the case that a theory choice function will lead to an intransitive (or incomplete) all-things-considered ranking. For example, pairwise majority voting is minimally rational with respect to at least some profiles in $\mathfrak{D}_3^3$. Suppose that the scientific virtues of accuracy, simplicity and scope provide the following profile of preference rankings over $\mathfrak{T} = \{x, y, z\} : \langle x \prec_{si} z \prec_{si} y, x \prec_{ac} z \prec_{ac} y, x \prec_{sc} y \prec_{sc} z\rangle$, and that x, y and z and simplicity, accuracy and scope exhaust the alternatives and virtues under consideration. Then majority voting yields the all-things-considered ranking of $x \prec z \prec y$. But this is only one profile for which majority voting delivers a transitive complete ranking. And given Arrow's theorem we know there is at least one for which it will fail to aggregate into a collective preference ranking. What the concept of minimal rationality is intended to highlight is that the number of profiles for which pairwise majority voting is 'successful' is relevant for whether it is rational or not to employ it. We calculate the exact numbers of profiles that pairwise majority voting maps to transitive all-things-considered rankings below, but for now consider what we may find after such an analysis. If the profile presented above were found to be the only profile in $\mathfrak{D}_3^3$ with respect to which majority voting were minimally rational, then scientists using this function would expect to succeed in rationally choosing in only 1 out of 216 of the possible cases.[5] Suppose, however, again in $\mathfrak{D}_3^3$ it was found that, there was only one profile of preferences which majority voting mapped to an intransitive ranking. In such a scenario, scientists would expect to succeed in making a rational choice using the function in 215 out of 216 of the possible cases.

In the above example, there is a sense in which pairwise majority would be *less* 'rational' (in an intuitive sense) if it generated a transitive ranking from only 1 out of 216 profiles, than it would be if it did so from 215. If a scientist used the function in the former case she would be acting 'irrationally.' But she wouldn't if she did so in the latter. In fact, not using pairwise majority in such a scenario would be 'irrationally' cautious.[6] Perhaps the scientist wants to know whether or not

[5]Assuming all 216 possible ways are equally likely to obtain, the chances of succeeding in rationally choosing the best theory are very low. See a discussion of this assumption further below.

[6]Some level of risk aversion is undoubtedly rational. Consider the following scenario: you are offered two bets. Bet 1 gives you the chance to win 100$ with probability 1. Bet 2 gives you a chance to win 200$ with probability .5 and 0$ otherwise. Choosing Bet 1 in this instance does not seem irrational, and in fact, many people will do so. However, as the probability of winning 200$ in Bet 2 increases, Bet 2 becomes more appealing, and fewer people will avoid it. There seems to be a

a.marcoci@lse.ac.uk

she should go through the rigmarole of generating rankings by virtues in order to choose between a given set of alternatives. If there were little hope that her preferred function mapped the resulting profile to a preference ranking, then this would be a waste of her time. But if there is a high chance that her function will deliver such a ranking and she wants to make a choice between the alternatives, then she should proceed.

So, whether or not a function is 'rational' seems sensitive to how likely it is to deliver a transitive and complete preference ranking. And how likely it is to deliver such a ranking, for a domain $\mathfrak{D}^n_m$, depends on the likelihood assigned to each of the profiles within that domain. This requires introducing a probability measure $\mathtt{Pr}$ over (the powerset of) $\mathfrak{D}^n_m$.[7] In the aforementioned discussion we assumed that $\mathtt{Pr}$ was the equiprobable distribution, with $\mathtt{Pr}$ assigning $1/(n!)^m$ to each profile in $\mathfrak{D}^n_m$.[8] But suppose, for comparison, that the probability of the single profile which mapped to an intransitive ranking in the latter example above were approaching 1. Then in that case the scientist would be 'irrational' to attempt to rank theories according to virtues.

This suggests the possibility of a degree measure of rationality. The degree to which an aggregation function f is rational (for a given $\mathfrak{D}^n_m$, $\mathtt{Pr}$) is simply the sum of the values $\mathtt{Pr}$ assigns to all profiles with respect to which f is minimally-rational. We denote this sum by μ and we call the resulting notion of rationality, rationality by degrees.

Rationality by degrees A theory choice function f, which meets UD, WP, IIA and ND (with respect to a domain, $\mathfrak{D}^n_m$) is μ-rational (or rational to degree μ), with respect to $\mathtt{Pr}$ if and only if

$$\mathtt{Pr}(\{\mathcal{P} \in \mathfrak{D}^n_m | f \text{ is minimally rational for } \mathcal{P}\}) = \mu$$

The shift from thinking about rationality in an all-or-nothing sense, to thinking about it in degrees is done in two steps. Firstly, we introduce a probability function $\mathtt{Pr}$ over the powerset of $\mathfrak{D}^n_m$. Secondly, we measure the rationality of a theory choice function f by the probability mass assigned to the set of all profiles in $\mathfrak{D}^n_m$ with respect to which f is minimally rational. Some correspondences between the notions emerge. If f is 1-rational, then it is rational for $\mathfrak{D}^n_m$, or equivalently, minimally

point at which choosing Bet 1 over Bet 2 becomes irrationally cautious (if this still doesn't appeal to your intuition, consider Bet 3 with probability .9 of winning 1,000,000$ and 99$ otherwise). In this sense, a scientist refraining from using pairwise majority, as this rule fails in 1 out of 216 cases appears irrationally cautious.

[7]The only restriction we place on $\mathtt{Pr}$ is that it assigns a non-zero probability to every profile in $\mathfrak{D}^n_m$. The motivation for this restriction is the same as the motivation for UD. If a theory choice function is rational only if it is defined over every profile in a domain, then all profiles are considered as 'live options'. Assigning to any profile a zero probability of occurring would undermine this.

[8]In the social choice literature this is known as the 'impartial culture' assumption Gehrlein (1983). This assumption is discussed in more detail below.

a.marcoci@lse.ac.uk

Table 26.1 The μ-rationality of pairwise majority for n theories and m virtues

		Theories		
		3	4	5
Virtues	3	.94444	.8298	.67573
	5	.93055	.7896	
	7	.92490		
	9	.92202		

rational with respect to every $\mathcal{P} \in \mathfrak{D}^n_m$. If, on the other hand, f is 0-rational, then f is minimally irrational with respect to every $\mathcal{P} \in \mathfrak{D}^n_m$.[9]

How μ-rational is pairwise majority then? We investigate the degree of rationality for certain values of m and n for a probability function, `Pr` assigning equal weight to all elements of $\mathfrak{D}^n_m$.[10] Table 26.1 summarizes these results.

The values in Table 26.1 indicate that pairwise majority becomes less rational as one increases the numbers of theories under consideration and the number of virtues used to evaluate them.[11] The same trend can be observed when calculating the probability of a Condorcet winner under pairwise majority voting. A Condorcet winner is an alternative which is all-things-considered preferred to each other alternative in pairwise comparisons. Notice that even if an all-things-considered ranking is intransitive there may be still one alternative which is better than all other, i.e. the cycle occurs lower in the ranking. The likelihood of a Condorcet winner has already been investigated in a series of papers in the social choice literature, i.e. Gehrlein and Fishburn (1976) and Gehrlein (1983). Table 26.2 collects some of the results of multiple papers.[12]

So what can we learn from Tables 26.1 and 26.2? In cases in which scientists are choosing among a small number of theories, the values remain quite elevated. For instance in choosing between three theories based on five virtues, pairwise majority is .9306-rational. In other words, in less than 7% of cases will a scientist trying to use majority voting run into an intransitive all-things-considered ranking. So, refraining from eliciting the individual rankings of theories based on virtues on account of Arrow's result is irrationally cautious in this scenario. Nevertheless, as scientists choose between increasing numbers of alternatives, using increasing numbers of virtues, the μ-rationality of pairwise majority decreases. Finding a precise threshold for when μ is high enough to warrant starting the aggregation procedure, or low enough to refrain from doing so, is not our focus here. It suffices to note that the

[9]These correspondences rely on our restriction on `Pr` stated in Footnote 7 above.

[10]The values in Table 26.1 have been calculated in *Mathematica 10*. Please contact the authors if you wish to consult the notebooks used.

[11]For a more sophisticated discussion (in the context of social choice) of the results in this table, as well as for a general formula for approximating the probability of a cycle given any number of voters (odd) and any number of alternatives, see DeMeyer and Plott (1970).

[12]The reason for this is that performing these calculations is a computational demanding task and some of the older papers did not have the technical means of obtaining all results.

a.marcoci@lse.ac.uk

Table 26.2 The μ-Condorcet-rationality of pairwise majority for n theories and m virtues

Theories													
		3	4	5	6	7	8	9	10	11	12	13	14
Virtues	3	.94444	.8888	.8399	.7977	.7612	.7293	.7011	.6760	.6536	.6333	.6148	.5980
	5	.93055	.8611	.80048	.74865	.70424	.66588	.63243		.57682		.53235	
	7	.92498	.84997	.78467	.72908	.68168	.64090	.60551	.64090	.54703		.50063	
	9	.92202	.84405	.77628	.71873	.66976		.59135		.534		.486	

a.marcoci@lse.ac.uk

μ-rationality of a theory choice function is sensitive to the numbers of alternatives and virtues under consideration. For relatively low numbers of both, theory choice using majority voting is rational *enough*.

26.6 Discussion

In this section we address some possible objections to Okasha's framework and our analysis in this paper.

26.6.1 Kuhn vs. Okasha

According to Kuhn, scientists guide their choice of theories by looking at how those theories fare with respect to a series of virtues, such as simplicity, accuracy, scope, etc. Okasha interprets this claim as saying that each of these virtues induces a complete ranking over the set of theories and that each scientist aggregates (according to an algorithm set a priori) all of these rankings into an all-things-considered ranking. This then models the order in which the scientist endorses the theories under consideration. We take Okasha's challenge to be that, per Arrow's impossibility theorem, such an aggregation cannot be guaranteed a priori. That is, prior to eliciting the individual rankings, a scientist cannot be sure that the algorithm chosen to aggregate them will deliver an ordering. In consequence, it seems that a scientist wishing to decide what theory to endorse based on this 'Kuhnian' procedure is irrational. In this paper we argue that there are plausible theory choice situations in which a scientist would appear irrationally cautious not to employ this Kuhnian procedure as long as the algorithm she uses is pairwise majority voting.

But one could question Okasha's interpretation of Kuhn's ideas on multi-criterial theory choice. Firstly, Kuhn does not say that scientific virtues induce complete rankings over the set of alternatives. Secondly, he does not construe theory choice as an algorithmic decision from a set of individual rankings into an all-things-considered ranking. Thirdly, Kuhn does not talk about scientists having a complete all-things-considered ranking over the set of theories. These considerations suggest that Okasha's reinterpretation of Kuhn's project in the context of social choice theory builds in significant assumptions concerning the nature of theory choice. But the purpose of this article is not to engage in Kuhnian hermeneutics, but rather to reply to Okasha's challenge to the rationality of theory choice. And whether an algorithmic procedure for arriving at an all-things considered best theory is possible is an interesting, albeit less Kuhnian than Okasha sells it to be, question. This paper shows that the viability of such an algorithm hinges on allowing an all-or-nothing vs. a degrees view of scientific rationality and on setting a threshold for what counts as rational suitable for the theory choice situation one is facing.

a.marcoci@lse.ac.uk

26.6.2 Impartial Culture

In articulating the notion of rationality by degrees we remarked that we require a probability distribution defined over the space of all possible profiles definable over *m* virtues and *n* theories. There we assumed that this probability distribution is the equiprobable one. In the social choice literature this kind of assumption is known as the 'impartial culture' (IC) assumption (Gehrlein 1983). Impartial cultures are a natural starting point: they make computations much easier and they have been widely studied in the social choice literature.[13]

But our primary motivation for assuming the equiprobable distribution is epistemic.[14] Prior to beginning the process of eliciting the rankings according to each virtue, a scientist cannot deem how likely it is for a particular profile to obtain. Therefore, from the perspective of the scientist, IC functions as a principle of indifference with respect to the different ways virtues rank the competing theories. Of course if one were to assume a different probability distribution, then one would expect the probability of majority cycles occurring to change. But as far as we can see, there is no reason to assume that a different probability distribution (for example, one where different virtues were less likely to submit the same ranking as one another than is assumed in IC) would *increase* the probability of majority cycles. Moreover, it's difficult to see how such a probability distribution could be justified from the *ex ante* perspective of the scientist, before she has elicited the rankings of the theories by virtues.

26.6.3 How Many Alternatives?

Returning to the problem of theory choice, the numbers presented in Tables 26.1 and 26.2 suggest that if a theory choice situations is placed in the upper left corner – 3 to 5 virtues and 3 to 5 theories, then the threat of Okasha's Arrovian result is quite small. And consequently, moving to a degree notion of rationality would save the rationality of theory choice (at least in that choice situation). But as we are moving away from the upper left corner, and especially if we are increasing the number of theories under consideration, the likelihood of a theory choice situation becomes so low that one can no longer make the claim that theory choice is rational enough.

If this model is supposed to capture the kind of theory choice Kuhn was considering, i.e. the choice between different paradigms, then it should only care

[13] Interestingly, it has been proven that as the number of voters tends to infinity, any deviation from impartial culture will reduce the probability of majority cycles, as long as the deviation isn't in favour of a distribution that assumes the Condorcet paradox from the start (Tsetlin et al. 2003). However, since we are working in a context with a finite, relatively small number of virtues (voters) we cannot rely on this result to motivate impartial culture here.

[14] We are grateful to an anonymous referee for encouraging us to motivate this assumption.

a.marcoci@lse.ac.uk

about very few theories. However, most of the theory choice situations Kuhn discusses are binary, such as the move from the Ptolemaic to the Copernican model of the Solar system or the shift from Newtonian to relativistic physics. These are situations in which Arrow's theorem does not create any problems. Okasha identifies this as a challenge and suggests that, in fact, we should be interested in more realistic theory choice situations such as:

> statistical estimation, where a researcher might want to estimate the value of a real-valued parameter in the unit interval; the alternatives that must be chosen between are uncountably many. So focusing exclusively on binary choice, as a way of trying to avoid the Arrovian predicament, is at odds with scientific practice. (p. 95)

Although we have no way of gauging how many alternatives are usually in play in theory choice situations, the scenario Okasha sketches above is not one that we consider worrisome for the results in this paper. The reason for this is simple. The choice of a real-value parameter is not a multi-criterial one. Such a choice is one in which there is a single criterion: accuracy.[15] Nothing changes in terms of simplicity, scope, etc. when a real-valued parameter is assigned a different value.

26.7 Conclusion

This paper contrasted the view that scientific rationality is an all-or-nothing notion with the view that scientific rationality comes in degrees. We showed that the choice between these two views can have significant implications to how rational we think theory choice is in the face of Okasha's Arrovian challenge.

There may be some for whom the mere possibility, irrespective of how small, of the virtues leaving them without an ordering is enough to make them doubt the possibility of using the virtues to select between competing theories. To them we can only respond that the purpose of this paper was to gauge the threat Okasha raised and evaluate what the prospects of rational theory choice remain in the aftermath of applying Arrow's impossibility theorem to theory choice.

References

Arrow, K. 1951. *Social choice and individual values*. New York: John Wiley.

Bradley, S. 2016. Constraints on rational theory choice. *British Journal for the Philosophy of Science*. https://academic.oup.com/bjps/article/doi/10.1093/bjps/axv063/2669744/Constraints-on-Rational-Theory-Choice

[15] We are grateful to an anonymous reviewer for pointing out that if this involves Bayesian statistical model selection then the criterion might not be accuracy, but rather posterior probability. However, as Okasha (2011,pp.105–110) notes such an approach to model selection is immune from the Arrovian challenge.

a.marcoci@lse.ac.uk

DeMeyer, F., and C.R. Plott. 1970. The probability of a cyclical majority. *Econometrica* 38(2): 345–354.

Gaertner, W., and N. Wüthrich. 2016. Evaluating competing theories via a common language of qualitative verdicts. *Synthese* 193: 3293–3309.

Gehrlein, W. 1983. Condorcet's paradox. *Theory and Decision* 15(2): 161–197.

Gehrlein, W., and P. Fishburn. 1976. The probability of the paradox of voting: A computable solution. *Journal of Economic Theory* 13: 14–25.

Kuhn, T. 1972. Objectivity, value judgment, and theory choice. In *The essential tension*. Chicago: University of Chicago Press.

List, C. 2013. Social choice theory. In *The Stanford Encyclopedia of Philosophy*, ed. E.N. Zalta, Winter 2013 edition. https://plato.stanford.edu/archives/win2013/entries/social-choice/

Morreau, M. 2014. Mr. Fit, Mr. Simplicity and Mr. Scope: From social choice to theory choice. *Erkenntnis* 79(6): 1253–1268.

Okasha, S. 2011. Theory choice and social choice: Kuhn versus Arrow. *Mind* 120(477): 83–115.

Rizza, D. 2013. Arrow's theorem and theory choice. *Synthese* 191(8): 1–10.

Stegenga, J. 2015. Theory choice and social choice: Okasha versus Sen. *Mind* 124(493): 263–277.

Tsetlin, I., M. Regenwetter, and B. Grofman. 2003. The impartial culture maximizes the probability of majority cycles. *Social Choice and Welfare* 21(3): 387–398.

a.marcoci@lse.ac.uk